

# One-loop Corrections to the $S$ Parameter in the Four-site Model

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## Abstract

We compute the leading chiral-logarithmic corrections to the  $S$  parameter in the four-site Higgsless model. In addition to the usual electroweak gauge bosons of the Standard Model, this model contains two sets of heavy charged and neutral gauge bosons. In the continuum limit, the latter gauge bosons can be identified with the first excited Kaluza-Klein states of the  $W^\pm$  and  $Z$  bosons of a warped extra-dimensional model with an  $SU(2)_L \times SU(2)_R \times U(1)_X$  bulk gauge symmetry. We consider delocalized fermions and show that the delocalization parameter must be considerably tuned from its tree-level *ideal* value in order to reconcile experimental constraints with the one-loop results. Hence, the delocalization of fermions does not solve the problem of large contributions to the  $S$  parameter in this class of theories and significant contributions to  $S$  can potentially occur at one-loop.

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## I. INTRODUCTION

As the world awaits the turn-on of the Large Hadron Collider (LHC) at CERN, theorists and experimentalists alike are left to ponder the question of the source of electroweak symmetry breaking (EWSB). In the Standard Model (SM), the electroweak symmetry is spontaneously broken by a single  $SU(2)$  scalar doublet which acquires a non-zero vacuum expectation value (vev) and, subsequently, gives masses to the SM gauge bosons. However, once radiative corrections are included, the physical Higgs boson mass is found to be quadratically divergent and extreme fine-tuning is required in order to achieve a mass on the order of hundreds of GeV. This has become known as the *large hierarchy problem*. Supersymmetric extensions of the SM can reduce the magnitude of fine-tuning due to the presence of new particles in the loops contributing to the Higgs boson mass.

Alternatively, warped extra-dimensional (or Randall-Sundrum (RS)) models propose to solve the large hierarchy problem by embedding the SM in an extra-dimensional setup [1]. In these models, the electroweak scale is generated from a large scale (i.e., the Planck scale) through an exponential hierarchy. The original version of the RS model contained a slice of  $AdS_5$  space bounded by two boundaries (or branes) where the SM was assumed to live on one of the boundaries. Motivated by the AdS/CFT correspondence [2, 3, 4], more recent versions of the RS scenario consider matter fields and fermions of the SM propagating in the bulk, while the Higgs is constrained to live on (or very near) the IR brane. In order to avoid large tree-level corrections to the  $\rho$  parameter in these scenarios, one must extend the bulk gauge group to a left-right symmetric form ( $SU(2)_L \times SU(2)_R \times U(1)$ ) [5]. Finally, it has been shown that the source of electroweak symmetry breaking in these models need not come from a fundamental scalar field. In fact, by imposing certain boundary conditions on the gauge fields, one can give masses to the SM gauge bosons. These models have been dubbed *Higgsless models* [6, 7, 8, 9].

In the gauge sector of these models, one expects a massless photon, light SM-like gauge bosons ( $W^\pm$  and  $Z^0$ ) plus towers of Kaluza-Klein (KK) partners to the light SM-like gauge bosons. Thus, a strong constraint on these models comes from considering the  $S$  parameter (since  $S$  effectively “counts” the degrees of freedom in the electroweak sector)[10, 11]. In fact, the tree-level contributions to  $S$  in these models can be quite large providing strong constraints on the KK gauge boson masses. However, by judiciously choosing the localization

of the light fermions in the bulk, the large tree-level corrections to  $S$  can be cancelled [12, 13, 14, 15, 16, 17, 18, 19]. In light of this, it becomes imperative to assess higher-order corrections to  $S$  in these models.

As in the SM, the one-loop corrections to gauge-boson self-energies in these models can be split into several gauge-invariant (and  $R_\xi$  gauge-independent) pieces. In other words, one can consider the effects of new fermions, the Higgs boson (or other scalars) and the KK gauge bosons separately. The effects of new fermions on the  $S$  parameter in these models have been studied in Refs. [20, 21, 22], while the corrections from the Higgs sector have recently been calculated in Ref. [23] using a “holographic” approach. However, the effect of one-loop corrections to  $S$  from loops of gauge bosons in a model with a bulk gauge symmetry  $SU(2)_L \times SU(2)_R \times U(1)$  remains unknown.

The corrections to gauge-boson self-energies from loops of gauge bosons suffer from the fact that the final answer depends on the particular  $R_\xi$  gauge that one uses to define the propagators of the gauge bosons circulating in the loop. This is a well-known problem that appears even in the SM [24]. The remedy for this situation is to extract other gauge-dependent terms from vertex and box corrections and combine these with the corrections to the two-point functions such that all  $R_\xi$  gauge-dependent terms are cancelled. This process is known as the *pinch technique* [25, 26, 27, 28]. It quickly becomes apparent that, in models with extra gauge bosons in addition to the SM ones, the computation of the  $S$  parameter can be quite complicated. Recently, however, a systematic algorithm for computing the one-loop corrections to  $S$  from extra gauge bosons has been developed in Ref. [29].

In this paper, we consider the effects of KK gauge bosons on the  $S$  parameter by studying the analogous effects in the deconstructed four-site model [30, 31]. These models are generalizations of the BESS model with two new triplets of gauge bosons[32, 33]. The one-loop corrections to  $S$  in the three-site model [34, 35, 36] have recently been computed in Refs. [29, 37, 38] and a fit to the electroweak data has been performed in Ref. [39]. The four-site model is based on a linear moose diagram with a gauge structure of  $[SU(2)]^3 \times U(1)$ . The gauge groups are linked together via non-linear sigma model fields  $\Sigma_i$  with link constants  $f_i$ . Once the electroweak symmetry is broken, the gauge sector of this model consists of a massless photon, light SM-like gauge bosons ( $W^\pm$  and  $Z^0$ ) plus two sets of heavier gauge bosons ( $\rho_i^\pm$  and  $\rho_i^0$  with  $i = 1, 2$ ). In the continuum limit, the latter can be thought of as the first excited KK states of the SM  $W^\pm$  and  $Z^0$ . Additionally, by choosing different values for

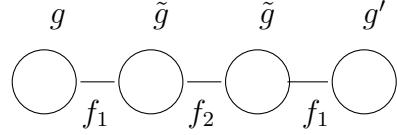


FIG. 1: The moose diagram for the four-site model.

the various link constants ( $f_i$ ), one can mimic the warped nature of the extra dimension in RS-type models.

The paper is structured in the following way. In Section II, we describe the model and calculate the mass eigenvalues and eigenvectors for both the charged and neutral gauge bosons. In Section III, we present the chiral logarithmic corrections to the gauge boson self energies from gauge boson self interactions and the resulting contribution to the  $S$  parameter is given in Section IV. We conclude with some observations in Section V.

## II. THE MODEL

In this section, we describe in detail the four-site model [30, 31]. The model is based on an  $SU(2)_L \times SU(2)_{V_1} \times SU(2)_{V_2} \times U(1)$  gauge symmetry and is depicted by the moose diagram shown in Fig. 1. The corresponding gauge fields are  $L^\mu$ ,  $V_1^\mu$ ,  $V_2^\mu$ , and  $R^\mu$ , with gauge couplings  $g$ ,  $\tilde{g}$ ,  $\tilde{g}$  and  $g'$  respectively.

The Lagrangian for the model consists of several parts. First, the non-linear sigma model terms are given by:

$$\mathcal{L}_{n\ell\sigma m} = \sum_{i=1}^3 \frac{f_i^2}{4} \left[ \text{Tr} D^\mu \Sigma_i D_\mu \Sigma_i^\dagger \right], \quad (1)$$

where the covariant derivatives are defined as:

$$D_\mu \Sigma_1 = \partial_\mu \Sigma_1 - ig L_\mu \Sigma_1 + i\tilde{g} \Sigma_1 V_{1\mu} \quad (2)$$

$$D_\mu \Sigma_2 = \partial_\mu \Sigma_2 - i\tilde{g} V_{1\mu} \Sigma_2 + i\tilde{g} \Sigma_2 V_{2\mu} \quad (3)$$

$$D_\mu \Sigma_3 = \partial_\mu \Sigma_3 - i\tilde{g} V_{2\mu} \Sigma_3 + ig' \Sigma_3 R_\mu, \quad (4)$$

with:

$$L_\mu = T^a L_\mu^a ; \quad V_{i\mu} = T^a V_{i\mu}^a ; \quad R_\mu = T^3 B_\mu, \quad (5)$$

and  $T^a = \frac{\sigma^a}{2}$ , where  $\sigma^a$  are the usual Pauli matrices. Note that, in general, the link constants  $f_i$  are free to take any value. In this paper, however, we will consider two distinct

possibilities: one where all  $f_i$  are equal and one where the middle link constant takes a different value from the other two (which are set equal to each other). In the continuum limit, the former corresponds to a flat extra-dimension, while the latter corresponds to a warped extra-dimension.

The gauge-boson kinetic terms are given by:

$$\mathcal{L}_g = -\frac{1}{2} \text{Tr}[L^{\mu\nu}]^2 - \frac{1}{2} \sum_{i=1}^2 \text{Tr}[V_i^{\mu\nu}]^2 - \frac{1}{2} \text{Tr}[R^{\mu\nu}]^2 \quad (6)$$

where  $L_{\mu\nu}$ ,  $V_i^{\mu\nu}$  and  $R^{\mu\nu}$  are the matrix field-strengths of the four gauge groups.

Next, we consider the couplings of light fermions to the various gauge groups. With the fermions completely localized to the two end sites, the four-site model generates a large correction to the  $S$  parameter at tree-level. However, if one allows the fermions to have small, non-zero couplings to the interior sites, this large tree-level contribution to  $S$  can be cancelled completely. It has been shown in Refs. [17, 30] that, in general, it is enough to consider one-site delocalization in order to cancel the large tree-level contributions. Thus, we assume that the light fermions couple mainly to the two end groups, as well as a small coupling to  $SU(2)_{V_1}$  such that the Lagrangian takes the form:

$$\mathcal{L}_f = -g' \bar{\psi} \gamma_\mu (Y_L P_L + Y_R P_R) B^\mu \psi - g(1 - x_1) \bar{\psi} \gamma_\mu T^a L^{a,\mu} P_L \psi - \tilde{g} x_1 \bar{\psi} \gamma_\mu T^a V_1^{a,\mu} P_L \psi, \quad (7)$$

where  $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$ , the electromagnetic charge is related to the isospin,  $Q_{em} = T_3 + Y$  and the parameter  $x_1$  measures the amount of delocalization and is assumed to be  $0 < x_1 \ll 1$ . We note that this expression is not separately gauge-invariant under  $SU(2)_L$  and  $SU(2)_{V_1}$ . Rather, the fermions should be viewed as being charged under  $SU(2)_L$  and the terms proportional  $x_1$  arise from an operator of the form:

$$\mathcal{L}'_f = -x_1 \bar{\psi} \gamma^\mu (i D_\mu \Sigma_1 \Sigma_1^\dagger) P_L \psi. \quad (8)$$

In addition to the terms listed above, one must also include higher-derivative operators since the theory is non-renormalizable. In particular, several  $\mathcal{O}(p^4)$  operators that one can write down are relevant to the  $S$  parameter. Expressing these in terms of the four-site model gauge fields, the relevant operators are [34]:

$$\mathcal{L}_4 = c_1 \tilde{g} g' \text{Tr}[V_{2,\mu\nu} \Sigma_3 B^{\mu\nu} T^3 \Sigma_3^\dagger] + c_2 \tilde{g} g \text{Tr}[V_{1,\mu\nu} \Sigma_1^\dagger L^{\mu\nu} \Sigma_1] + c_3 \tilde{g}^2 \text{Tr}[V_{2,\mu\nu} \Sigma_2^\dagger V_1^{\mu\nu} \Sigma_2]. \quad (9)$$

Finally, Ref. [31] has shown that by including an  $L_{10}$ -like mixing between the middle two sites:

$$\mathcal{L}_\epsilon = -\frac{\epsilon}{2} \text{Tr}[V_{1,\mu\nu}\Sigma_2 V_{2,\mu\nu}\Sigma_2^\dagger], \quad (10)$$

one can cancel the dangerously large tree-level contributions to the  $S$  parameter without delocalizing the light fermions. To avoid ghosts, one must require that the free parameter  $\epsilon$  satisfy  $|\epsilon| < 1$ . However, one finds that the value of  $\epsilon$  required to cancel the  $S$  parameter at tree-level is of order one which is much larger than would be expected from naive dimensional analysis [40, 41]. In the following, therefore, we will neglect this term and study the simpler version of the four site model given by the sum of Eqs. (1), (6), (7) and (9).

The model approximates the SM in the limit:

$$x \equiv \frac{g}{\tilde{g}} \ll 1, \quad y \equiv \frac{g'}{\tilde{g}} \ll 1, \quad (11)$$

in which case we expect the spectrum in the gauge sector to consist of a massless photon, light SM-like  $W$  and  $Z$  bosons and two sets of heavy bosons which we denote as  $\rho_i^\pm$  and  $\rho_i^0$  with  $i = 1, 2$ . The four-site model couplings  $g$  and  $g'$  are then numerically equal to the SM  $SU(2)_L$  and  $U(1)_Y$  gauge couplings respectively in this limit. We therefore define an angle  $\theta$  such that:

$$g^2 \simeq \frac{4\pi\alpha}{s^2} = \frac{e^2}{s^2}, \quad g'^2 \simeq \frac{4\pi\alpha}{c^2}, \quad \frac{s}{c} = \frac{g'}{g}, \quad (12)$$

where  $s(c) = \sin \theta(\cos \theta)$ ,  $\alpha$  is the fine-structure constant and  $e$  is the charge of the electron.

### A. Mass Eigenstates and Their Interactions

In unitary gauge (where  $\Sigma_i \equiv \mathcal{I}$ ), the quadratic piece of the full Lagrangian gives rise to mass terms for the neutral ( $M_{NC}$ ) and charged ( $M_{CC}$ ) gauge bosons of the form:

$$\mathcal{L} = \frac{1}{2} \sum_{i=0}^3 W_{i,\mu} M_{ij,NC}^2 W_j^\mu + \sum_{i=0}^2 W_{i,\mu} M_{ij,CC}^2 W_j^\mu, \quad (13)$$

where, in “site” space, the vectors  $W_{i,\mu}$  are given by:

$$\begin{pmatrix} L_\mu \\ V_{1,\mu} \\ V_{2,\mu} \\ B_\mu \end{pmatrix}. \quad (14)$$

The eigenstates corresponding to Eq. (13) satisfy the eigenvalue equation:

$$M^2 \vec{v}_n = m_n^2 \vec{v}_n , \quad (15)$$

where  $\vec{v}_n$  is a vector in site space with components  $v_n^i$ . The superscript  $i$  labels the sites, running from 0 to 2 for the charged bosons ( $n = W^\pm, \rho_1^\pm, \rho_2^\pm$ ), and 0 to 3 for neutral ones ( $n = A, Z^0, \rho_1^0, \rho_2^0$ ). Then, choosing eigenvectors normalized by  $\vec{v}_n^T \vec{v}_m = \delta_{mn}$ , the gauge eigenstates ( $W_\mu^i$ ) and mass eigenstates ( $W'_{n\mu}$ ) are related by:

$$W_\mu^i = \sum_n v_n^i W'_{n\mu} . \quad (16)$$

### 1. The Charged Sector

First, we consider the charged gauge boson sector. The mass matrix in this sector takes the form:

$$\mathcal{M}_{CC}^2 = \frac{\tilde{g}^2}{4} \begin{pmatrix} x^2 f_1^2 & -x f_1^2 & 0 \\ -x f_1^2 & f_1^2 + f_2^2 & -f_2^2 \\ 0 & -f_2^2 & f_1^2 + f_2^2 \end{pmatrix} . \quad (17)$$

Since we are interested in the limit  $x = g/\tilde{g} \ll 1$ , we diagonalize the mass matrices perturbatively in  $x$ . To  $\mathcal{O}(x^2)$ , we find the mass of the SM-like  $W$  boson is:

$$M_W^2 \simeq \frac{g^2}{4} \frac{f_1^2 f_2^2}{f_1^2 + 2f_2^2} \left( 1 - x^2 z_W \right) , \quad (18)$$

while the masses of the heavier charged gauge bosons are:

$$M_{\rho_1^\pm}^2 \simeq \frac{\tilde{g}^2 f_1^2}{4} \left( 1 + \frac{x^2}{2} \right) , \quad (19)$$

and:

$$M_{\rho_2^\pm}^2 \simeq \frac{\tilde{g}^2 (f_1^2 + 2f_2^2)}{4} \left( 1 + \frac{x^2}{2} z^4 \right) , \quad (20)$$

where:

$$z = \frac{f_1}{\sqrt{f_1^2 + 2f_2^2}} , \quad (21)$$

$$z_W = \frac{f_1^4 + 2f_1^2 f_2^2 + 2f_2^4}{(f_1^2 + 2f_2^2)^2} = \frac{1}{2} (1 + z^4) . \quad (22)$$

For future reference, we note that the ratios of the masses are given by (for small values of  $x^2$ ):

$$\begin{aligned}\frac{M_W^2}{M_{\rho_1^\pm}^2} &\simeq x^2 \left( \frac{1-z^2}{2} \right), \\ \frac{M_W^2}{M_{\rho_2^\pm}^2} &\simeq x^2 \left( \frac{z^2(1-z^2)}{2} \right), \\ \frac{M_{\rho_1^\pm}^2}{M_{\rho_2^\pm}^2} &\simeq z^2 \left( 1 + \frac{x^2}{2}(1-z^4) \right).\end{aligned}\tag{23}$$

Finally, we expand the gauge-eigenstate fields in terms of the mass eigenstates as:

$$L_\mu^\pm = v_{W^\pm}^L W_\mu^\pm + v_{\rho_1^\pm}^{L\pm} \rho_{1,\mu}^\pm + v_{\rho_2^\pm}^{L\pm} \rho_{2,\mu}^\pm,\tag{24}$$

$$V_{1,\mu}^\pm = v_{W^\pm}^{V_1} W_\mu^\pm + v_{\rho_1^\pm}^{V_1\pm} \rho_{1,\mu}^\pm + v_{\rho_2^\pm}^{V_1\pm} \rho_{2,\mu}^\pm,\tag{25}$$

$$V_{2,\mu}^\pm = v_{W^\pm}^{V_2} W_\mu^\pm + v_{\rho_1^\pm}^{V_2\pm} \rho_{1,\mu}^\pm + v_{\rho_2^\pm}^{V_2\pm} \rho_{2,\mu}^\pm.\tag{26}$$

We give the explicit expressions for the eigenvector components in Appendix A.

## 2. The Neutral Sector

The mass matrix for the neutral gauge fields takes the form:

$$\mathcal{M}_{NC}^2 = \frac{\tilde{g}^2}{4} \begin{pmatrix} x^2 f_1^2 & -x f_1^2 & 0 & 0 \\ -x f_1^2 & f_1^2 + f_2^2 & -f_2^2 & 0 \\ 0 & -f_2^2 & f_1^2 + f_2^2 & -x t f_1^2 \\ 0 & 0 & -x t f_1^2 & x^2 t^2 f_1^2 \end{pmatrix},\tag{27}$$

where  $t \equiv \tan \theta$ . Again, we diagonalize the mass matrix perturbatively in  $x$ . We find one zero eigenvalue corresponding to the photon as well as three massive states:

$$M_Z^2 \simeq \frac{g^2}{4c^2} \frac{f_1^2 f_2^2}{f_1^2 + 2f_2^2} \left( 1 - x^2 z_Z \right),\tag{28}$$

$$M_{\rho_1^0}^2 \simeq \frac{\tilde{g}^2 f_1^2}{4} \left( 1 + \frac{x^2}{2c^2} \right),\tag{29}$$

$$M_{\rho_2^0}^2 \simeq \frac{\tilde{g}^2 (f_1^2 + 2f_2^2)}{4} \left( 1 + \frac{x^2 z^4}{2c^2} \right),\tag{30}$$

where:

$$z_Z = \frac{1}{2} \frac{(z^4 + \cos^2 2\theta)}{\cos^2 \theta}. \quad (31)$$

Next, we expand the gauge-eigenstate fields in terms of the mass eigenstates as:

$$L_\mu^3 = v_A^L A_\mu + v_Z^L Z_\mu + v_{\rho_1^0}^L \rho_{1,\mu}^0 + v_{\rho_2^0}^L \rho_{2,\mu}^0, \quad (32)$$

$$V_{1,\mu}^3 = v_A^{V_1} A_\mu + v_Z^{V_1} Z_\mu + v_{\rho_1^0}^{V_1} \rho_{1,\mu}^0 + v_{\rho_2^0}^{V_1} \rho_{2,\mu}^0, \quad (33)$$

$$V_{2,\mu}^3 = v_A^{V_2} A_\mu + v_Z^{V_2} Z_\mu + v_{\rho_1^0}^{V_2} \rho_{1,\mu}^0 + v_{\rho_2^0}^{V_2} \rho_{2,\mu}^0, \quad (34)$$

$$B_\mu = v_A^B A_\mu + v_Z^B Z_\mu + v_{\rho_1^0}^B \rho_{1,\mu}^0 + v_{\rho_2^0}^B \rho_{2,\mu}^0. \quad (35)$$

The expressions for the individual  $v_i^j$ 's are given in Appendix A.

### III. ONE-LOOP CORRECTIONS TO THE GAUGE BOSON SELF-ENERGIES

In this section, we compute the one-loop corrections needed to calculate the  $S$  parameter in the four-site model. The  $S$  parameter is defined in the mass eigenstate basis as [11]:

$$\frac{\alpha \Delta S}{4s^2 c^2} = \Delta \Pi_{ZZ}(M_Z^2) - \Delta \Pi_{AA}(M_Z^2) - \frac{c^2 - s^2}{cs} \Delta \Pi_{ZA}(M_Z^2), \quad (36)$$

where:

$$\Delta \Pi_{ij}(M_Z^2) \equiv \frac{\Pi_{ij}(M_Z^2) - \Pi_{ij}(0)}{M_Z^2}, \quad (37)$$

and our convention for the self-energies is:

$$i\Pi_{ij}^{\mu\nu}(q^2) = g^{\mu\nu} \Pi_{ij}(q^2) + (q^\mu q^\nu \text{ term}). \quad (38)$$

Since the four-site model is a non-renormalizable theory, the one-loop corrections will result in expressions which are divergent. In other words, calculating the one-loop corrections using dimensional regularization (in  $d = 4 - 2\epsilon$  dimensions) the resulting expressions contain terms which diverge as  $1/\epsilon$ . Alternatively, if we were to compute the one-loop corrections using a momentum cutoff, the divergent terms are logarithms of the form  $\log \frac{\Lambda^2}{M^2}$  where  $\Lambda$  is assumed to be the cutoff scale of the effective theory and  $M$  is the heaviest of the masses circulating in the loop. If the hierarchy between  $\Lambda$  and  $M$  is large, then the contributions from these *chiral logarithms* dominates over any finite terms. Below, we will compute the leading chiral-logarithmic contributions to the  $S$  parameter in the four-site model.

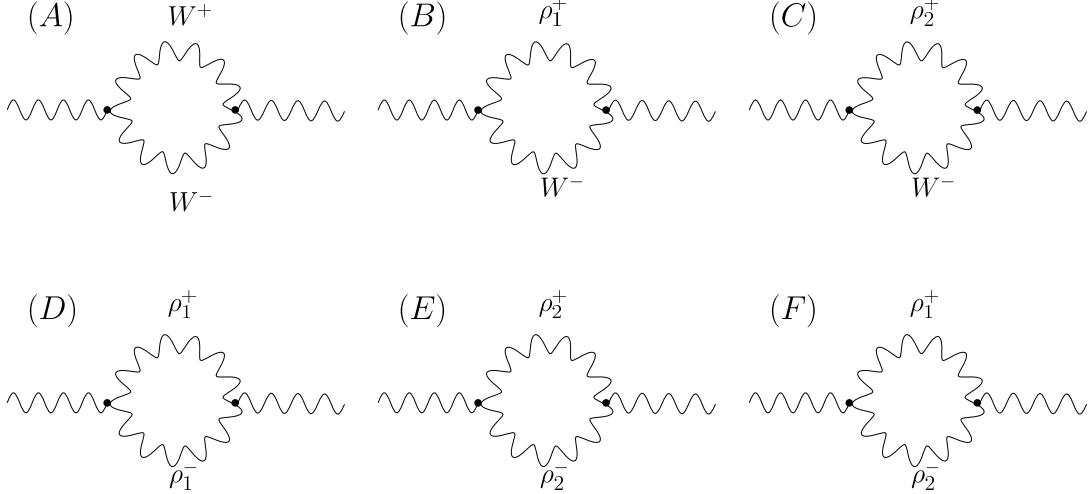


FIG. 2: One-loop corrections to the two-point functions  $\Pi_{AA}$ ,  $\Pi_{ZZ}$  and  $\Pi_{ZA}$  in the four-site model.

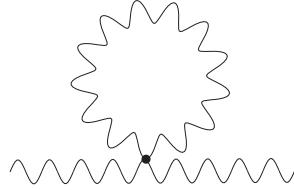


FIG. 3: Diagram containing four-point couplings between gauge bosons.

### A. The Two-point Functions

The diagrams which contribute to the gauge-boson two-point functions in the four-site model are shown in Fig. 2. Note that diagrams which contain four-point interactions such as Fig. 3 do not contain any  $q^2$ -dependence (where  $q$  is the momentum of the external gauge bosons). Hence, given Eq. (37), these diagrams do not contribute to  $S$  and we will neglect them in the following. Note also that we are working in unitary gauge where only physical particles contribute to the loops.

The individual contributions from the diagrams in Fig. 2 to the gauge-boson two-point functions are summarized in Appendix C. Summing the contributions to the photon two-point function, we find:

$$\Delta\Pi_{AA}^{two-pt.} = \frac{\alpha}{4\pi} \left[ \left( 7 - \frac{7}{6c^2} - \frac{1}{12c^4} \right) \log \frac{\Lambda^2}{M_W^2} + 7 \log \frac{\Lambda^2}{M_{\rho_1^\pm}^2} + 7 \log \frac{\Lambda^2}{M_{\rho_2^\pm}^2} \right]. \quad (39)$$

The total contribution to the  $Z$ -photon mixing amplitude  $\Delta\Pi_{ZA}$  is:

$$\Delta\Pi_{ZA}^{two-pt.} = \frac{\alpha}{4\pi cs} \left[ c^2 \left( 7 - \frac{7}{6c^2} - \frac{1}{12c^4} \right) \log \frac{\Lambda^2}{M_W^2} + \frac{7(c^2 - s^2)}{2} \log \frac{\Lambda^2}{M_{\rho_1^\pm}^2} + \frac{7(c^2 - s^2)}{2} \log \frac{\Lambda^2}{M_{\rho_2^\pm}^2} \right]. \quad (40)$$

Finally, the total contribution to the  $Z$  boson two-point function from the diagrams in Fig. 2 is:

$$\begin{aligned} \Delta\Pi_{ZZ}^{two-pt.} = & \frac{\alpha}{4\pi s^2 c^2} \left[ c^4 \left( 7 - \frac{7}{6c^2} - \frac{1}{12c^4} \right) \log \frac{\Lambda^2}{M_W^2} \right. \\ & + \left[ \frac{17}{24} \frac{(1-z^4)^2}{(1-z^2)} + \frac{7(c^2 - s^2)^2}{4} \right] \log \frac{\Lambda^2}{M_{\rho_1^\pm}^2} \\ & \left. + \left[ \frac{17}{24} z^2(1+z^4) + \frac{25}{12} z^4 + \frac{7(c^2 - s^2)^2}{4} \right] \log \frac{\Lambda^2}{M_{\rho_2^\pm}^2} \right]. \end{aligned} \quad (41)$$

## B. Pinch Contributions and the Total Self-energies

As discussed earlier, the one-loop corrections to the two-point functions from loops of gauge bosons exhibit non-trivial dependence on the particular  $R_\xi$  gauge used to define the gauge boson propagators. This gauge-dependence carries over into the calculation of observables such as the oblique parameters resulting in gauge-dependent expressions for  $S$ ,  $T$  and  $U$ . The remedy for this situation is to isolate gauge-dependent terms from other one-loop corrections (i.e., vertex and box corrections) and combine these with the one-loop corrections to the two-point functions. The result is a gauge-independent expression which can reliably be compared to experimental data. This technique, known as the Pinch Technique (PT), was first developed for the SM, but, recently, an algorithm has been developed to extend the PT to theories with extra gauge bosons beyond those of the SM [29]. This algorithm was utilized to compute  $S$  and  $T$  at one-loop in the three-site model and shown to produce identical results to those of Refs. [37, 38] which were obtained using different methods. In this section, we compute the “pinch” contributions in the four-site model.

The vertex corrections which give rise to pinch contributions are shown in Fig. 4. We note that contributions involving internal neutral gauge bosons cancel among themselves (see Eq. (58) of Ref. [29]). In deriving the fermion-gauge boson couplings, we assume that the delocalization parameter,  $x_1$ , is of  $\mathcal{O}(x^2)$  (a necessary condition to cancel the tree level contribution to  $S$ ). Using the results of Ref. [29], we find the chiral logarithmic contributions

from the vertex pinch diagrams are to  $\mathcal{O}(x^0)$ :

$$\Delta\Pi_{AA}^{vertex} = \frac{\alpha}{4\pi} \left[ \frac{8}{3c^2} + \frac{1}{6c^4} \right] \log \frac{\Lambda^2}{M_W^2}, \quad (42)$$

$$\Delta\Pi_{ZA}^{vertex} = \frac{\alpha}{4\pi sc} \left[ c^2 \left( \frac{4}{3c^2} + \frac{1}{12c^4} \right) \log \frac{\Lambda^2}{M_W^2} - \frac{3}{4}(1+z^2) \left( 1 - \frac{x_1}{x^2} \right) \log \frac{\Lambda^2}{M_{\rho_1^\pm}^2} \right], \quad (43)$$

$$\Delta\Pi_{ZZ}^{vertex} = \frac{\alpha}{4\pi s^2 c^2} \left[ -\frac{3}{2} c^2 (1+z^2) \left( 1 - \frac{x_1}{x^2} \right) \right] \log \frac{\Lambda^2}{M_{\rho_1^\pm}^2}. \quad (44)$$

The box corrections which contribute to the pinch pieces are shown in Fig. 5. Summing the individual diagrams, we find the total contributions are:

$$\Delta\Pi_{AA}^{box} = \frac{\alpha}{4\pi} \left[ -\frac{3}{2c^2} - \frac{1}{12c^4} \right] \log \frac{\Lambda^2}{M_W^2}, \quad (45)$$

$$\Delta\Pi_{ZA}^{box} = 0, \quad (46)$$

$$\Delta\Pi_{ZZ}^{box} = \frac{\alpha}{4\pi s^2 c^2} \left( \frac{3c^2}{2} \right) \log \frac{\Lambda^2}{M_W^2}. \quad (47)$$

Finally, summing the two-point, vertex and box contributions, we find the gauge-independent PT self-energies are:

$$\Delta\Pi_{AA}^{PT} = \frac{\alpha}{4\pi} \left[ 7 \log \frac{\Lambda^2}{M_W^2} + 7 \log \frac{\Lambda^2}{M_{\rho_1^\pm}^2} + 7 \log \frac{\Lambda^2}{M_{\rho_2^\pm}^2} \right], \quad (48)$$

$$\begin{aligned} \Delta\Pi_{ZA}^{PT} = & \frac{\alpha}{4\pi sc} \left[ c^2 \left( 7 + \frac{1}{6c^2} \right) \log \frac{\Lambda^2}{M_W^2} \right. \\ & + \left( \frac{7(c^2 - s^2)}{2} - \frac{3}{4}(1+z^2) \left( 1 - \frac{x_1}{x^2} \right) \right) \log \frac{\Lambda^2}{M_{\rho_1^\pm}^2} \\ & \left. + \frac{7(c^2 - s^2)}{2} \log \frac{\Lambda^2}{M_{\rho_2^\pm}^2} \right], \end{aligned} \quad (49)$$

$$\begin{aligned} \Delta\Pi_{ZZ}^{PT} = & \frac{\alpha}{4\pi s^2 c^2} \left[ c^2 \left( 7c^2 + \frac{1}{3} - \frac{1}{12c^2} \right) \log \frac{\Lambda^2}{M_W^2} \right. \\ & + \left( \frac{17}{24} \frac{(1-z^4)^2}{(1-z^2)} + \frac{7(c^2 - s^2)^2}{4} - \frac{3}{2} c^2 (1+z^2) \left( 1 - \frac{x_1}{x^2} \right) \right) \log \frac{\Lambda^2}{M_{\rho_1^\pm}^2} \\ & \left. + \left( \frac{17}{24} z^2 (1+z^4) + \frac{25}{12} z^4 + \frac{7(c^2 - s^2)^2}{4} \right) \log \frac{\Lambda^2}{M_{\rho_2^\pm}^2} \right]. \end{aligned} \quad (50)$$

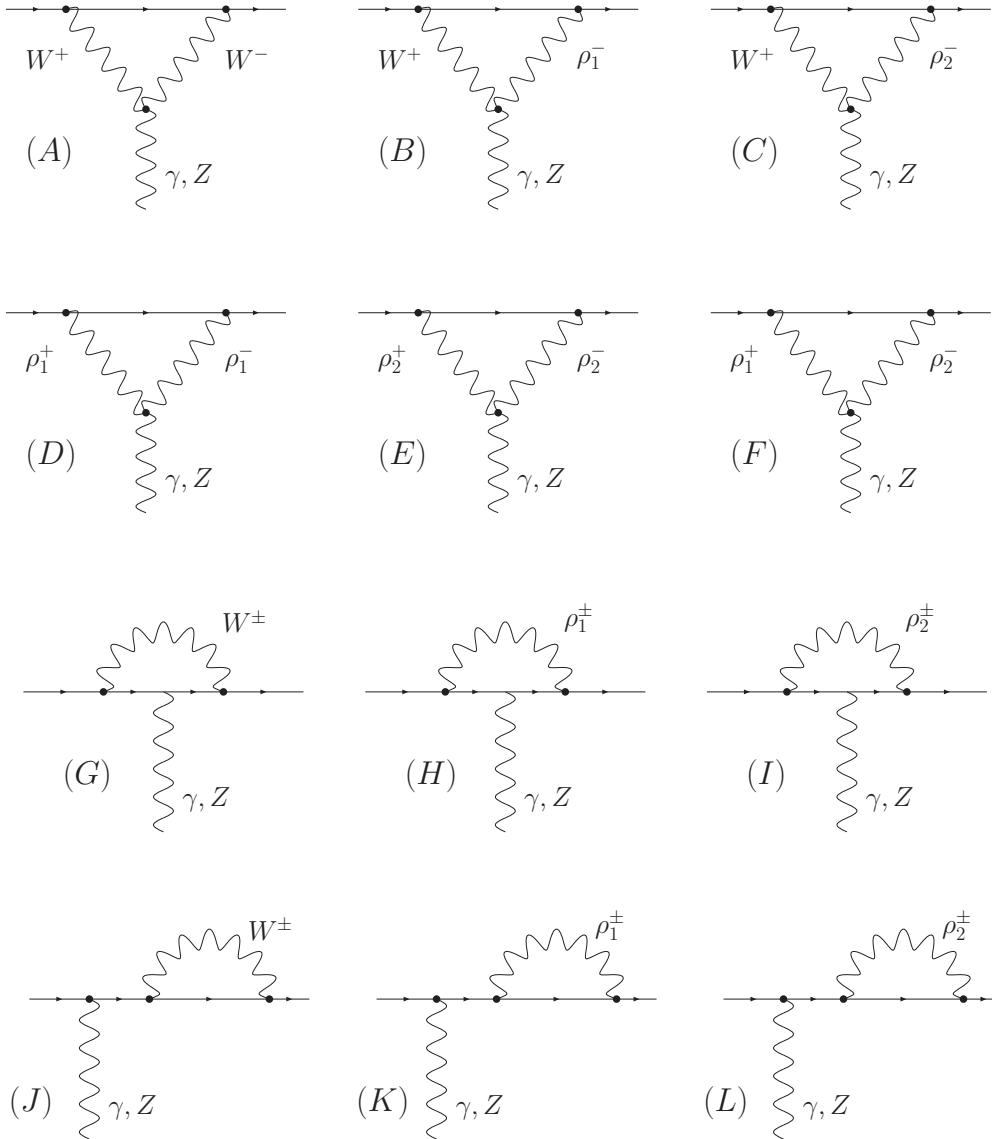


FIG. 4: One-loop vertex corrections which contribute to the gauge-invariant self-energies in the four-site model.

#### IV. THE $S$ PARAMETER AT ONE-LOOP

In this section, we compute the  $S$  parameter at the one-loop level in the four-site model. First, let us consider the contribution at tree-level. In general, the tree-level contribution to the  $S$  parameter from an  $SU(2)^{N+1} \times U(1)$  deconstructed Higgsless model with one-site

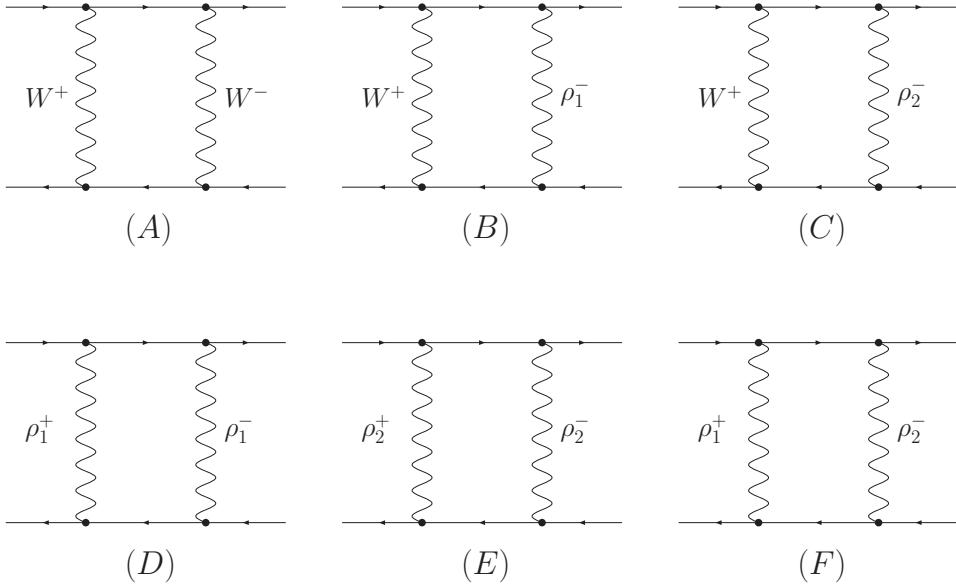


FIG. 5: One-loop box corrections which contribute to the gauge-invariant self-energies in the four-site model.

fermion delocalization is given by [17]:

$$\alpha S_{tree} = 4s^2 c^2 M_Z^2 \left( \sum_{i=1}^N \frac{1}{M_{\rho_i^0}^2} - x_1 \tilde{\mathbf{m}}^{-2} \right), \quad (51)$$

where  $\tilde{\mathbf{m}}$  is the (0,0) component of the neutral gauge boson mass matrix. In particular, for the four-site model, we have:

$$\begin{aligned} \alpha S_{tree} &= \frac{4s^2 M_W^2}{M_{\rho_1^0}^2} \left[ 1 + \frac{M_{\rho_1^0}^2}{M_{\rho_2^0}^2} - x_1 \left( \frac{4M_{\rho_1^0}^2}{g^2 f_1^2} \right) \right] \\ &\simeq \frac{4s^2 M_W^2}{M_{\rho_1^\pm}^2} \left( 1 + z^2 - \frac{x_1 M_{\rho_1^\pm}^2}{2M_W^2} (1 - z^2) \right) + \mathcal{O}(x^2), \end{aligned} \quad (52)$$

where  $x_1$  measures the amount of delocalization of the light fermions. It is easy to see that one can exactly cancel the large tree-level contribution to  $S$  if:

$$x_1 = \frac{2M_W^2(1+z^2)}{M_{\rho_1^\pm}^2(1-z^2)}. \quad (53)$$

This situation is termed *ideal delocalization* [18].

The one-loop corrections to  $S$  for delocalized fermions are given by substituting Eqs. (48)-(50) into Eq. (36). Doing this, we find:

$$\begin{aligned}\Delta S = & \frac{1}{12\pi} \log \frac{\Lambda^2}{M_W^2} - \left[ \frac{(43 + z^2 + 17z^4 + 17z^6)}{24\pi} - \frac{3}{4\pi} \frac{x_1}{x^2} (1 + z^2) \right] \log \frac{\Lambda^2}{M_{\rho_1^\pm}^2} \\ & - \left[ \frac{(42 - 17z^2 - 50z^4 - 17z^6)}{24\pi} \right] \log \frac{\Lambda^2}{M_{\rho_2^\pm}^2},\end{aligned}\quad (54)$$

which can be written in the more suggestive form:

$$\begin{aligned}\Delta S = & \frac{1}{12\pi} \log \frac{M_{\rho_1^\pm}^2}{M_W^2} - \left[ \frac{(41 + z^2 + 17z^4 + 17z^6)}{24\pi} - \frac{3}{4\pi} \frac{x_1}{x^2} (1 + z^2) \right] \log \frac{M_{\rho_2^\pm}^2}{M_{\rho_1^\pm}^2} \\ & - \left[ \frac{(83 - 16z^2 - 33z^4)}{24\pi} - \frac{3}{4\pi} \frac{x_1}{x^2} (1 + z^2) \right] \log \frac{\Lambda^2}{M_{\rho_2^\pm}^2}.\end{aligned}\quad (55)$$

We note that the first term in Eq. (55), which arises from scaling between  $M_W$  and  $M_{\rho_1^\pm}$ , has the same coefficient as the leading chiral-logarithmic contribution from a heavy Higgs boson:

$$S_{Higgs} = \frac{1}{12\pi} \log \frac{M_H^2}{M_W^2}. \quad (56)$$

This is expected, however, since the gauge- and chiral-symmetries of the four-site model in this energy range are the same as the SM with a heavy Higgs boson [11]. Since experimental limits on  $S$  assume the existence of a fundamental Higgs boson, in order to compare our prediction with data, we must subtract Eq. (56) from Eq. (55).

An important check on our calculation is provided by considering the limit  $f_2 \rightarrow \infty$  where the four-site model reduces to the three-site model [31]. In this limit, we see from Eq. (21) that  $z \rightarrow 0$  and from Eq. (20) that  $M_{\rho_2^\pm} \rightarrow \infty$ . In this situation, we identify  $M_{\rho_2^\pm}$  with the cutoff of the effective theory ( $\Lambda$ ) and Eq. (55) reduces to:

$$\Delta S = \frac{1}{12\pi} \log \frac{M_{\rho_1^\pm}^2}{M_W^2} - \left[ \frac{41}{24\pi} - \frac{3}{4\pi} \frac{x_1}{x^2} \right] \log \frac{\Lambda^2}{M_{\rho_1^\pm}^2}. \quad (57)$$

This expression is exactly the one obtained in Refs. [37, 38] for the three-site model (using two different methods and two different gauges) and checked numerically in Ref. [29]. We note that, originally, this term accounted for scaling between  $M_{\rho_1^\pm}$  and  $M_{\rho_2^\pm}$ , i.e., the energy range where the gauge- and chiral-symmetries of the four-site model are identical to those of the three-site model.

As mentioned in Section II, the  $S$  parameter also receives contributions from the dimension-4 operators of Eq. (9) [34]. By using Eqs. (32)-(35), these may be written as [37]:

$$\mathcal{L}_4|_{Z,A}^{quad} = \frac{i}{2}\delta_{ZZ}(Z_\mu D^{\mu\nu} Z_\nu) + i\delta_{ZA}(Z_\mu D^{\mu\nu} A_\nu) + \frac{i}{2}\delta_{AA}(A_\mu D^{\mu\nu} A_\nu), \quad (58)$$

where  $D^{\mu\nu} = -\partial^2 g^{\mu\nu} + \partial^\mu \partial^\nu$ . We find the  $\delta_{ij}$ 's in the four-site model take the form:

$$\delta_{ZZ} = \frac{e^2}{s^2 c^2} \left[ c_1 s^2 (z^2 - (c^2 - s^2)) + c_2 c^2 (z^2 + (c^2 - s^2)) - \frac{1}{2} c_3 (z^4 - (c^2 - s^2)^2) \right], \quad (59)$$

$$\delta_{ZA} = \frac{e^2}{2sc} \left[ c_1 (-z^2 + (c^2 - 3s^2)) + c_2 (z^2 + (3c^2 - s^2)) + 2c_3 (c^2 - s^2) \right] \quad (60)$$

$$\delta_{AA} = 2e^2 [c_1 + c_2 + c_3]. \quad (61)$$

Inserting these expressions into Eq. (36), we find the contribution to  $S$  from the dimension-4 operators is:

$$\delta S_{1-loop} = -8\pi [(1 - z^2)(c_1 + c_2) + (1 + z^4)c_3]. \quad (62)$$

An interesting question to ask in models with delocalized fermions is whether or not the *ideal* value of  $x_1$  which cancels the large tree-level contribution to the  $S$  parameter is *ideal* at higher-orders in perturbation theory. In other words, the question is whether or not  $x_1$  must be tuned order-by-order in perturbation theory in such a way to bring  $S$  into agreement with precision electroweak data.

To study this issue in the four-site model, we apply the following numerical analysis. The model initially has five input parameters: the three gauge couplings and the two link parameters, along with the cutoff,  $\Lambda$ . We have taken as our input parameters  $\alpha$ ,  $G_F$  and  $M_Z$  and computed the weak mixing angle as described in Appendix D. The two remaining parameters we choose as  $M_{\rho_1^\pm}$  and  $z$ . In Fig. 7, we plot the tree-level expression for  $S$  (Eq. (52)) as well as the sum of the tree-level and one-loop contributions (with  $c_1 = c_2 = 0$ ) as a function of  $x_1$ . In this plot, we have taken  $z = 0.58$  (corresponding to the flat  $f_2 = f_1$  scenario) and we have identified the Higgs reference mass with  $M_{\rho_1^\pm}$  and assumed two values of the cutoff scale. The horizontal dotted line in the plot denotes the value  $S = 0$ . As we can see, in going from tree-level to the one-loop level, the value of  $x_1$  must be tuned by factors of  $\sim 5$  or more depending on the value of  $\Lambda$  and there are potentially large cancellations

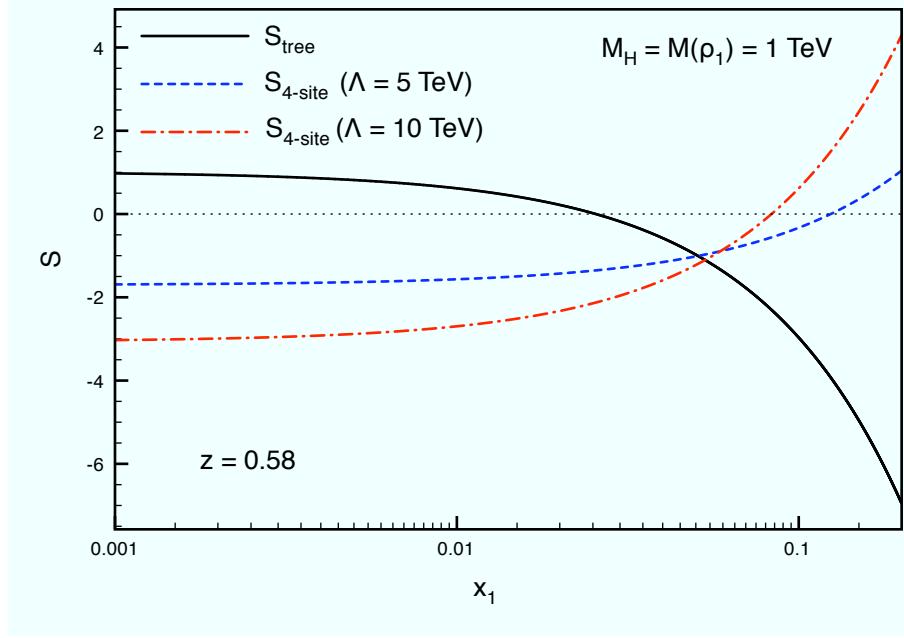


FIG. 6: The  $S$  parameter in the four-site model with delocalized fermions at the one-loop level as a function of the delocalization parameter  $x_1$ . In this plot,  $f_2 = f_1$ .

between the tree and the one-loop contributions. The result for a warped case ( $f_1 \neq f_2$ ) is shown in Fig. 6 and looks quite similar to the flat case.

Finally, summing Eqs. (52), (55) and (62), and accounting for the reference Higgs mass, we find our final result to be:

$$\begin{aligned}
\alpha S_{4-site} = & \left[ \frac{4s^2 M_W^2}{M_{\rho_1^\pm}^2} \left( 1 + z^2 - \frac{x_1 M_{\rho_1^\pm}^2}{2 M_W^2} (1 - z^2) \right) \right]_{\mu=\Lambda} + \frac{\alpha}{12\pi} \log \frac{M_{\rho_1^\pm}^2}{M_{H_{ref}}^2} \\
& - \frac{\alpha}{\pi} \left[ \frac{(41 + z^2 + 17z^4 + 17z^6)}{24} - \frac{3}{4} \frac{x_1}{x^2} (1 + z^2) \right] \log \frac{M_{\rho_2^\pm}^2}{M_{\rho_1^\pm}^2} \\
& - \frac{\alpha}{\pi} \left[ \frac{(83 - 16z^2 - 33z^4)}{24} - \frac{3}{4} \frac{x_1}{x^2} (1 + z^2) \right] \log \frac{\Lambda^2}{M_{\rho_2^\pm}^2} \\
& - 8\pi\alpha \left[ (1 - z^2)(c_1(\Lambda) + c_2(\Lambda)) + (1 + z^4)c_3(\Lambda) \right], \tag{63}
\end{aligned}$$

where the contributions from the tree-level and dimension-four operators are now understood to be evaluated at the scale  $\mu = \Lambda$ .

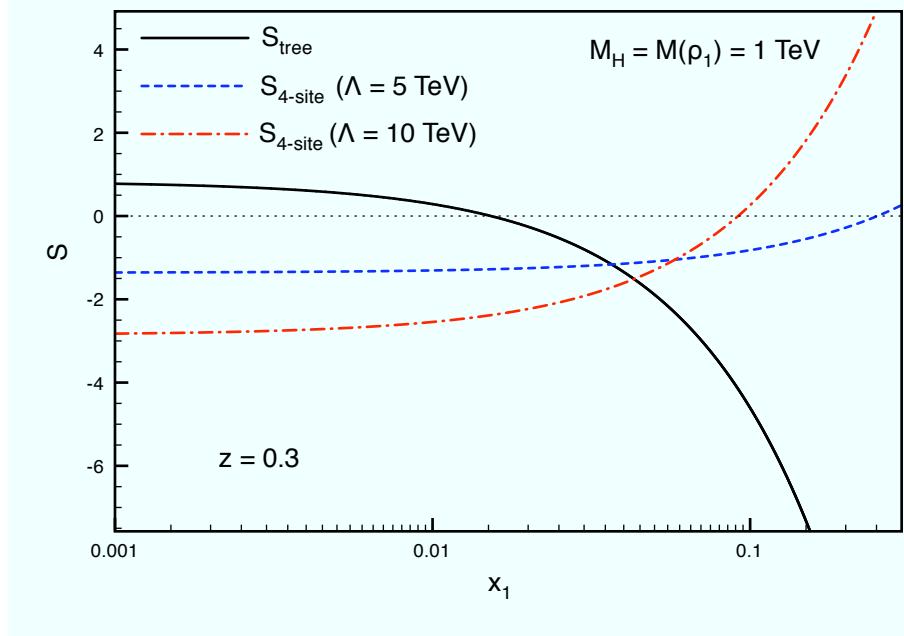


FIG. 7: The  $S$  parameter in the four-site model with delocalized fermions at the one-loop level as a function of the delocalization parameter  $x_1$ . In this plot,  $f_2 \neq f_1$ .

## V. DISCUSSION AND CONCLUSIONS

In this paper, we have computed the leading chiral-logarithmic corrections to the  $S$  parameter in the four-site model. The gauge sector of this model consists of a SM-like set of gauge bosons (massless photon and light vector gauge bosons  $W^\pm$  and  $Z$ ) plus two sets of heavier gauge bosons ( $\rho_i^\pm$  and  $\rho_i^0$  with  $i = 1, 2$ ). Thus, the spectrum is very similar to that of the lightest and next-to-lightest KK excitations of a Randall-Sundrum scenario with an  $SU(2)_L \times SU(2)_R \times U(1)_X$  bulk gauge symmetry [5].

Our results show that the  $S$  parameter in this model is UV-sensitive and therefore requires renormalization. This is similar to the situation in Ref. [23] where the one-loop corrections to  $S$  from the Higgs sector of a holographic model were computed and found to be logarithmically-divergent.

We find that the chiral-logarithmic corrections to  $S$  in the four-site model naturally separate into three distinct contributions: a model-independent piece arising from scaling from  $M_W$  up to the  $\rho_1^\pm$  mass, a piece which arises from scaling between  $M_{\rho_1^\pm}$  and  $M_{\rho_2^\pm}$  and a piece arising from scaling from  $M_{\rho_2^\pm}$  up to the cutoff of the effective theory ( $\Lambda$ ).

The coefficient of the model-independent term has the same form as the large Higgs-mass dependence of the  $S$  parameter in the SM. This allows us to compare our one-loop results directly with experimental constraints on  $S$ . Additionally, in the limit  $f_2 \rightarrow \infty$  where the four-site model reduces to the three-site model, we have shown that our results correctly reproduce the one-loop corrections to the  $S$  parameter in the three-site model [29, 37, 38].

In this work, we have focussed on the contributions to  $S$  at one-loop primarily from the gauge bosons of the model. In principle, there would be contributions from the extended fermion sector of the model as well. However, these contributions have been computed in the three-site model and shown to be negligible [37].

We have also studied the dependence of the one-loop results on the delocalization parameter  $x_1$ . In an *ideally-delocalized* situation, the large tree-level contribution to  $S$  present in the four-site model can be completely cancelled. The outstanding issue in these types of models, however, is whether or not  $x_1$  must be tuned order-by-order in perturbation theory to bring  $S$  into agreement with experimental constraints. We have shown that, in going from tree-level to the one-loop level, the *ideal* value of  $x_1$  must be tuned by a factor of 5 or more depending on the value of the cutoff scale  $\Lambda$ .

### Acknowledgements

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## APPENDIX A: EIGENVECTOR COMPONENTS

The eigenvector components for the charged gauge bosons (defined through Eqs. (24)-(26)) are<sup>1</sup>:

$$v_{W^\pm}^L \simeq 1 - \frac{x^2 z_W}{2}, \quad (\text{A1})$$

$$v_{\rho_1^\pm}^L \simeq -\frac{x}{\sqrt{2}} \left( 1 + \frac{x^2}{4} \frac{1 - 3z^2}{1 - z^2} \right), \quad (\text{A2})$$

$$v_{\rho_2^\pm}^L \simeq -\frac{x z^2}{\sqrt{2}} \left( 1 + \frac{x^2 z^2}{4} \frac{3z^4 - 5z^2 + 4}{1 - z^2} \right), \quad (\text{A3})$$

$$v_{W^\pm}^{V_1} \simeq \frac{(1 + z^2)x}{2} \left( 1 + \frac{x^2}{4} \frac{(1 - 3z^2)(1 + z^4)}{(1 + z^2)} \right), \quad (\text{A4})$$

$$v_{\rho_1^\pm}^{V_1} \simeq \frac{1}{\sqrt{2}} \left( 1 - \frac{x^2}{4} \frac{1 + z^2}{1 - z^2} \right), \quad (\text{A5})$$

$$v_{\rho_2^\pm}^{V_1} \simeq \frac{1}{\sqrt{2}} \left( 1 + \frac{x^2 z^4}{4} \frac{1 + z^2}{1 - z^2} \right), \quad (\text{A6})$$

$$v_{W^\pm}^{V_2} \simeq \frac{(1 - z^2)x}{2} \left( 1 + \frac{x^2}{4} (1 - 3z^4) \right), \quad (\text{A7})$$

$$v_{\rho_1^\pm}^{V_2} \simeq \frac{1}{\sqrt{2}} \left( 1 - \frac{x^2}{4} \frac{1 - 3z^2}{1 - z^2} \right), \quad (\text{A8})$$

$$v_{\rho_2^\pm}^{V_2} \simeq -\frac{1}{\sqrt{2}} \left( 1 - \frac{x^2 z^4}{4} \frac{3 - z^2}{1 - z^2} \right). \quad (\text{A9})$$

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<sup>1</sup> Our results agree with Ref. [30] except for Eq. A7, where we are a factor of  $\sqrt{2}$  larger, and Eq. A8, where our coefficient for the  $\mathcal{O}(x^2)$  term is a factor of 4 smaller.

For the neutral gauge bosons, the eigenvector components are<sup>2</sup>:

$$v_A^L \simeq s(1 - x^2 s^2), \quad (\text{A10})$$

$$v_Z^L \simeq c \left[ 1 - \frac{x^2}{4c^2} (1 + z^4 - 4s^4) \right], \quad (\text{A11})$$

$$v_{\rho_1^0}^L \simeq -\frac{x}{\sqrt{2}} \left[ 1 - \frac{x^2}{4c^2} \left( 1 - \frac{2(1-2z^2)\cos 2\theta}{1-z^2} \right) \right], \quad (\text{A12})$$

$$v_{\rho_2^0}^L \simeq -\frac{xz^2}{\sqrt{2}} \left[ 1 + \frac{x^2 z^2}{4c^2} \left( 2 - 3z^2 + \frac{2\cos 2\theta}{1-z^2} \right) \right], \quad (\text{A13})$$

$$v_A^{V_1} \simeq xs(1 - x^2 s^2), \quad (\text{A14})$$

$$v_Z^{V_1} \simeq \frac{x(z^2 + \cos 2\theta)}{2c} \left[ 1 - \frac{x^2}{4c^2} \left( \frac{3z^6 - \cos^3 2\theta - (1+2s^2)z^4 - (1-4c^2)z^2}{(z^2 + \cos 2\theta)} \right) \right], \quad (\text{A15})$$

$$v_{\rho_1^0}^{V_1} \simeq \frac{1}{\sqrt{2}} \left[ 1 - \frac{x^2}{4c^2} \left( 1 + \frac{2z^2 c^2}{1-z^2} \right) \right], \quad (\text{A16})$$

$$v_{\rho_2^0}^{V_1} \simeq \frac{1}{\sqrt{2}} \left[ 1 - \frac{x^2 z^4}{4c^2} \left( 1 - \frac{2\cos 2\theta}{1-z^2} \right) \right], \quad (\text{A17})$$

$$v_A^{V_2} \simeq xs(1 - x^2 s^2), \quad (\text{A18})$$

$$v_Z^{V_2} \simeq -\frac{x(z^2 - \cos 2\theta)}{2c} \left[ 1 - \frac{x^2}{4c^2} \left( \frac{3z^6 - (1+2c^2)z^4 + \cos^3 2\theta - (1-4s^4)z^2}{z^2 - \cos 2\theta} \right) \right], \quad (\text{A19})$$

$$v_{\rho_1^0}^{V_2} \simeq \frac{1}{\sqrt{2}} \left[ 1 - \frac{x^2}{4c^2} \left( 1 - \frac{2z^2 \cos 2\theta}{1-z^2} \right) \right], \quad (\text{A20})$$

$$v_{\rho_2^0}^{V_2} \simeq -\frac{1}{\sqrt{2}} \left[ 1 - \frac{x^2 z^4}{4c^2} \left( 1 + \frac{2\cos 2\theta}{1-z^2} \right) \right], \quad (\text{A21})$$

$$v_A^B \simeq c(1 - x^2 s^2), \quad (\text{A22})$$

$$v_Z^B \simeq -s \left[ 1 - \frac{x^2}{4c^2} (1 + z^4 - 4c^4) \right], \quad (\text{A23})$$

$$v_{\rho_1^0}^B \simeq -\frac{xt}{\sqrt{2}} \left[ 1 - \frac{x^2}{4c^2} \left( 1 + \frac{2(1-2z^2)\cos 2\theta}{1-z^2} \right) \right], \quad (\text{A24})$$

$$v_{\rho_2^0}^B \simeq \frac{xz^2 t}{\sqrt{2}} \left[ 1 + \frac{x^2 z^2}{4c^2} \left( 2 - 3z^2 - \frac{2\cos 2\theta}{1-z^2} \right) \right]. \quad (\text{A25})$$

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<sup>2</sup> Our results agree with Ref. [30].

## APPENDIX B: TRIPLE GAUGE BOSON INTERACTIONS AND COUPLINGS

Expanding the non-Abelian interaction terms of Eq. (6), we find the triple gauge boson interactions take the form:

$$\begin{aligned}\mathcal{L}_{AAA} = & ig \left[ (\partial_\mu L_\nu^+ - \partial_\nu L_\mu^+) L^{\mu-} L^{\nu 3} + \partial_\mu L_\nu^3 L^{\mu+} L^{\nu-} \right] \\ & + i\tilde{g} \sum_i^2 \left[ (\partial_\mu V_{i,\nu}^+ - \partial_\nu V_{i,\mu}^+) V_i^{\mu-} V_i^{\nu 3} + \partial_\mu V_{i,\nu}^3 V_i^{\mu+} V_i^{\nu-} \right] + \text{h.c.} .\end{aligned}\quad (\text{B1})$$

Inserting the expansions of the gauge eigenstate fields in terms of the mass eigenstate fields (Eqs. (24)-(26) and (32)-(34)) into this expression yields:

$$\begin{aligned}\mathcal{L}_{AAA} = & i \left\{ \sum_{n=A,Z,\rho_1^0,\rho_2^0} g_{W^+W^-}^n \left( W_{\mu\nu}^+ W^{\mu-} n^\nu + \frac{1}{2} n_{\mu\nu} W^{+\mu} W^{-\nu} \right) \right. \\ & + g_{W^+\rho_1^-}^n \left( (W_{\mu\nu}^+ \rho_1^{\mu-} + \rho_{1,\mu\nu}^+ W^{\mu-}) n^\nu + \frac{1}{2} n_{\mu\nu} (W^{\mu+} \rho_1^{\nu-} + \rho_1^{\mu+} W^{\nu-}) \right) \\ & + g_{W^+\rho_2^-}^n \left( (W_{\mu\nu}^+ \rho_2^{\mu-} + \rho_{2,\mu\nu}^+ W^{\mu-}) n^\nu + \frac{1}{2} n_{\mu\nu} (W^{\mu+} \rho_2^{\nu-} + \rho_2^{\mu+} W^{\nu-}) \right) \\ & + g_{\rho_1^+\rho_2^-}^n \left( (\rho_{1,\mu\nu}^+ \rho_2^{\mu-} + \rho_{2,\mu\nu}^+ \rho_1^{\mu-}) n^\nu + \frac{1}{2} n_{\mu\nu} (\rho_1^{\mu+} \rho_2^{\nu-} + \rho_2^{\mu+} \rho_1^{\nu-}) \right) \\ & + g_{\rho_1^+\rho_1^-}^n \left( \rho_{1,\mu\nu}^+ \rho_1^{\mu-} n^\nu + \frac{1}{2} n_{\mu\nu} \rho_1^{+\mu} \rho_1^{-\nu} \right) \\ & \left. + g_{\rho_2^+\rho_2^-}^n \left( \rho_{2,\mu\nu}^+ \rho_2^{\mu-} n^\nu + \frac{1}{2} n_{\mu\nu} \rho_2^{+\mu} \rho_2^{-\nu} \right) \right\} + \text{h.c.},\end{aligned}\quad (\text{B2})$$

where  $n_{\mu\nu} = \partial_\mu n_\nu - \partial_\nu n_\mu$  and  $n = A, Z, \rho_i, \rho_2$ . Using Eq. (D2), the couplings between three mass-eigenstate fields are expressed by using the wavefunctions from Appendix A as (to

	$n = A$	$n = Z$
$g_{W^+W^-}^n$	$e$	$\frac{ec}{s}$
$g_{\rho_1^+\rho_1^-}^n$	$e$	$\frac{e(c^2-s^2)}{2cs}$
$g_{\rho_2^+\rho_2^-}^n$	$e$	$\frac{e(c^2-s^2)}{2cs}$
$g_{W^+\rho_1^-}^n$	0	$-\frac{ex}{sc} \left( \frac{\sqrt{2}(1-z^4)}{4} \right)$
$g_{W^+\rho_2^-}^n$	0	0
$g_{\rho_1^+\rho_2^-}^n$	0	$\frac{ez^2}{2sc}$

TABLE I: The three-point couplings among the gauge fields in the four-site model to lowest-order in  $x$ . We have computed, but do not display, the corrections up to  $\mathcal{O}(x^3)$ .

$\mathcal{O}(x^2)$ :

$$g_{W^+W^-}^n = \frac{e}{s}(1+s^2x^2) \left[ (v_W^L)^2 v_n^L + \frac{1}{x} \left( (v_W^{V_1})^2 v_n^{V_1} + (v_W^{V_2})^2 v_n^{V_2} \right) \right] \quad (\text{B3})$$

$$g_{W^+\rho_1^-}^n = \frac{e}{s}(1+s^2x^2) \left[ v_W^L v_{\rho_1^\pm}^L v_n^L + \frac{1}{x} \left( v_W^{V_1} v_{\rho_1^\pm}^{V_1} v_n^{V_1} + v_W^{V_2} v_{\rho_1^\pm}^{V_2} v_n^{V_2} \right) \right] \quad (\text{B4})$$

$$g_{W^+\rho_2^-}^n = \frac{e}{s}(1+s^2x^2) \left[ v_W^L v_{\rho_2^\pm}^L v_n^L + \frac{1}{x} \left( v_W^{V_1} v_{\rho_2^\pm}^{V_1} v_n^{V_1} + v_W^{V_2} v_{\rho_2^\pm}^{V_2} v_n^{V_2} \right) \right] \quad (\text{B5})$$

$$g_{\rho_1^+\rho_2^-}^n = \frac{e}{s}(1+s^2x^2) \left[ v_{\rho_1^\pm}^L v_{\rho_2^\pm}^L v_n^L + \frac{1}{x} \left( v_{\rho_1^\pm}^{V_1} v_{\rho_2^\pm}^{V_1} v_n^{V_1} + v_{\rho_1^\pm}^{V_2} v_{\rho_2^\pm}^{V_2} v_n^{V_2} \right) \right] \quad (\text{B6})$$

$$g_{\rho_1^+\rho_1^-}^n = \frac{e}{s}(1+s^2x^2) \left[ (v_{\rho_1^\pm}^L)^2 v_n^L + \frac{1}{x} \left( (v_{\rho_1^\pm}^{V_1})^2 v_n^{V_1} + (v_{\rho_1^\pm}^{V_2})^2 v_n^{V_2} \right) \right] \quad (\text{B7})$$

$$g_{\rho_2^+\rho_2^-}^n = \frac{e}{s}(1+s^2x^2) \left[ (v_{\rho_2^\pm}^L)^2 v_n^L + \frac{1}{x} \left( (v_{\rho_2^\pm}^{V_1})^2 v_n^{V_1} + (v_{\rho_2^\pm}^{V_2})^2 v_n^{V_2} \right) \right]. \quad (\text{B8})$$

The couplings between three mass eigenstate gauge bosons are summarized in Table I.

## APPENDIX C: FEYNMAN GRAPH RESULTS

In this appendix, we present the chiral logarithmic contribution to  $\mathcal{O}(x^0)$  from each diagram in Fig. 2 which contributes to the two-point functions  $\Delta\Pi_{AA,ZA,ZZ}^{two-pt}$ . The SM contributions in unitary gauge can be found in the appendix of Ref. [42].

## 1. Photon Two-Point Amplitude $\Delta\Pi_{AA}^{two-pt.}$

The non-zero amplitudes which contribute to  $\Delta\Pi_{AA}$  are:

$$(\Delta\Pi_{AA})_A = \frac{\alpha}{4\pi} \left[ 7 - \frac{7}{6c^2} - \frac{1}{12c^4} \right] \log \frac{\Lambda^2}{M_W^2} \quad (C1)$$

$$(\Delta\Pi_{AA})_D = \frac{\alpha}{4\pi} (7) \log \frac{\Lambda^2}{M_{\rho_1^\pm}^2} \quad (C2)$$

$$(\Delta\Pi_{AA})_E = \frac{\alpha}{4\pi} (7) \log \frac{\Lambda^2}{M_{\rho_2^\pm}^2}, \quad (C3)$$

where we have neglected terms of  $\mathcal{O}(x^2)$ .

## 2. Z-Photon Mixing Amplitude $\Delta\Pi_{ZA}^{two-pt.}$

The non-zero amplitudes which contribute to  $\Delta\Pi_{ZA}^{two-pt.}$  are:

$$(\Delta\Pi_{ZA})_A = \frac{\alpha}{4\pi cs} \left[ c^2 \left( 7 - \frac{7}{6c^2} - \frac{1}{12c^4} \right) \right] \log \frac{\Lambda^2}{M_W^2} \quad (C4)$$

$$(\Delta\Pi_{ZA})_D = \frac{\alpha}{4\pi cs} \left( \frac{7(c^2 - s^2)}{2} \right) \log \frac{\Lambda^2}{M_{\rho_1^\pm}^2} \quad (C5)$$

$$(\Delta\Pi_{ZA})_E = \frac{\alpha}{4\pi cs} \left( \frac{7(c^2 - s^2)}{2} \right) \log \frac{\Lambda^2}{M_{\rho_2^\pm}^2}. \quad (C6)$$

### 3. $Z$ Boson Two-Point Amplitude $\Delta\Pi_{ZZ}^{two-pt.}$

The non-zero contributions to  $\Delta\Pi_{ZZ}^{two-pt.}$  are:

$$(\Delta\Pi_{ZZ})_A = \frac{\alpha}{4\pi s^2 c^2} \left[ c^4 \left( 7 - \frac{7}{6c^2} - \frac{1}{12c^4} \right) \right] \log \frac{\Lambda^2}{M_W^2} \quad (\text{C7})$$

$$(\Delta\Pi_{ZZ})_B = \frac{\alpha}{4\pi s^2 c^2} \left[ \frac{17}{24} \frac{(1-z^4)^2}{(1-z^2)} \right] \log \frac{\Lambda^2}{M_{\rho_1^\pm}^2} \quad (\text{C8})$$

$$(\Delta\Pi_{ZZ})_D = \frac{\alpha}{4\pi s^2 c^2} \left( \frac{7(c^2-s^2)^2}{4} \right) \log \frac{\Lambda^2}{M_{\rho_1^\pm}^2} \quad (\text{C9})$$

$$(\Delta\Pi_{ZZ})_E = \frac{\alpha}{4\pi s^2 c^2} \left( \frac{7(c^2-s^2)^2}{4} \right) \log \frac{\Lambda^2}{M_{\rho_2^\pm}^2} \quad (\text{C10})$$

$$(\Delta\Pi_{ZZ})_F = \frac{\alpha}{4\pi s^2 c^2} \left[ \frac{17}{24} z^2 (1+z^4) + \frac{25}{12} z^4 \right] \log \frac{\Lambda^2}{M_{\rho_2^\pm}^2}. \quad (\text{C11})$$

## APPENDIX D: ELECTROWEAK PARAMETERS IN 4-SITE MODEL

In this appendix, we define the relations between the SM input parameters used in our numerical analysis and the four-site model parameters. First, we define the  $SU(2)$  and  $U(1)$  couplings as  $g$  and  $g'$  with  $\frac{s}{c} = \frac{g'}{g}$ . Next, we take as inputs  $e$ ,  $M_Z$ ,  $G_\mu$ :

$$\begin{aligned} \alpha &= \frac{e^2}{4\pi} = 137.035999679^{-1} \\ M_Z &= 91.1875 \text{ GeV} \\ G_F &= 1.166637 \times 10^{-5} \text{ GeV}^{-2}. \end{aligned} \quad (\text{D1})$$

We expand the definition of the electromagnetic coupling  $e$  in the four-site model in powers of  $x$  as:

$$\begin{aligned} \frac{1}{e^2} &\equiv \frac{1}{g^2} + \frac{2}{\tilde{g}^2} + \frac{1}{g'^2} \\ &= \frac{1}{g^2 s^2} \left( 1 + 2s^2 x^2 \right). \end{aligned} \quad (\text{D2})$$

The relationship between the  $W^\pm$  and  $Z$  boson masses is given by:

$$M_W^2 = c^2 M_Z^2 \left( 1 + x^2 (z_Z - z_W) \right) \quad (\text{D3})$$

Next, in order to derive a relation for  $G_F$ , we consider the coupling between the SM-like  $W^\pm$  and light fermions. Using, Eq. (7), we find:

$$\begin{aligned} g_L^W &= g \left[ v_W^L (1 - x_1) + \frac{x_1}{x} v_W^{V_1} \right] \\ &= g \left[ 1 - x^2 \left( \frac{3}{4} - \frac{z^4}{4} \right) \right], \end{aligned} \quad (\text{D4})$$

where, for simplicity, we have assumed ideal delocalization for the light fermions. Then, Eq. (D4) corresponds to a value for  $G_F$  of:

$$\sqrt{2}G_F = \frac{(g_L^W)^2}{4M_W^2} = \frac{g^2}{4M_W^2} \left[ 1 - x^2 \left( \frac{3}{2} - \frac{z^4}{2} \right) \right]. \quad (\text{D5})$$

Finally, we can calculate the “Z standard” weak mixing angle  $\theta_Z$ :

$$\begin{aligned} s_Z^2 c_Z^2 &= \frac{e^2}{4\sqrt{2}G_F M_Z^2} \\ &= s^2 c^2 + 2x^2 s^2 (c^2 - s^2) \left( c^2 - \frac{1}{4}(1 + z^4) \right), \end{aligned} \quad (\text{D6})$$

where  $s_Z(c_Z) = \sin \theta_Z(\cos \theta_Z)$ . The relationship between the weak mixing angle  $\theta_Z$  and the angle defined in Eq. (12) is then expressed as:

$$s_Z^2 = s^2 + \Delta ; \quad c_Z^2 = c^2 - \Delta \quad (\text{D7})$$

where:

$$\Delta \equiv 2x^2 s^2 \left( c^2 - \frac{1}{4}(1 + z^4) \right). \quad (\text{D8})$$

Therefore, we see that the difference between  $s^2$  and  $s_Z^2$  is of  $\mathcal{O}(x^2)$ .

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